ANALYTICAL INVESTIGATIONS OF LAMINAR

SEPARATIONS USING THE "CROCCO-LEES

MIXING PARAMETER" METHOD

NGR-37-002-028

ANALYTICAL INVESTIGATIONS OF LAMINAR SEPARATIONS USING THE "CROCCO-LEES MIXING PARAMETER" METHOD

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Prepared Under Grant Number NGR 37-002-028 by
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Stillwater, Oklahoma

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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I. INTRODUCTION

Project Objectives

The objective of the present research is twofold. The first objective is to further the theoretical understanding of supersonic flow separation through an analytical investigation. The second objective is to develop a semi-empirical correlation of the significant parameters found in the analytical study based on available experimental data. Such correlations are necessary to the completion of a method of predicting the pressure distribution on a surface in a separating and reattaching supersonic flow.

Analytic analysis. The first step in the analytical analysis was a comprehensive survey and evaluation of the existing literature in the field. The major part of this survey has been carried out and a summary of the material is given in this report. The evaluation of current literature will be continued throughout the course of the project. An intensive search for suitable supersonic boundary layer data is also currently underway.

Considerable effort has been devoted to determining the significant parameters and the proper combination of simplifying assumptions. At the date of this report this part of the analysis of the problem is incomplete, although encouraging progress has been made. Once parametization of the problem is more nearly complete, the chosen parameters and techniques will be mated with empirical correlations from what data is available, forming an integrated analysis of the problem.

Empirical correlations. Both supersonic and subsonic experimental data have been analyzed in attempts to obtain valid empirical correlations. Thus far the correlations obtained have been unsatisfactory due to the lack of empirical data of a suitable nature. These correlation attempts and the problems involved are discussed in subsequent sections of this report.

II. LITERATURE SURVEY (ANALYTICAL METHODS)

II-1. General Discussion

The presentation of most literature in this field followed nearly the same pattern. The integral momentum equations are used almost exclusively as the simplifying technique for handling the boundary layer equations. Most investigators choose to analyze the problem in steps; for example, 1) prior to separation, 2) separation to shock impingement, 3) impingement to reattachment and 4) after reattachment. The initial assumptions for handling the boundary layer equations throughout all regions, in general, stem from prior knowledge of attached flows.

The behavior of the velocity profiles within the separated region is qualitatively understood. However, these profiles have been handled in several ways analytically. Some prefer polynomials (as used by Karman-Pohlhausen), while others have adopted the Falkner-Skan, Stewartson reversed flow, or combinations of these profiles. Some of these profiles have inherent advantages in one region while they become unrealistic representations in the following region. References (1)* to (7) present much of the literature describing the different boundary layer profiles.

Experimental studies of this phenomenon have provided a model of the flow which is used for theoretical considerations. (See Figures 1-a and 1-b for a pictorial representation of two commonly encountered flow situations). The majority of the analytical treatments based on this flow model have been based on the Crocco-Lees (8) method.

The analysis prior to shock impingement has received the more sophisticated treatment, while the reattachment is generally lumped together by one or two additional assumptions and is then represented by a simple set of equations.

The remainder of this section is devoted to a discussion of four recent methods which have appeared. They represent the current refinements and incorporate much of the earlier literature in their development.

II-2. Crocco-Lees Method by Glick (9)

The Crocco-Lees (8) method is based upon the assumption that the parameters describing the boundary layer are dependent upon the rate of entrainment of fluid into the boundary layer from the external stream and that there exist certain universal correlation functions which relate these parameters.

An extension of the Crocco-Lees method was made by Glick (9). This study of separated and reattaching ergions of flow led to a physical model which incorporates the concept of the "dividing" streamline and uses experimental data to determine values for the significant parameters. According to this physical model, viscous momentum transport is the essential mechanism in the zone between separation and the beginning of reattachment, while the reattachment process is, on the contrary, an essentially inviscid process.

The Crocco-Lees method divides the flow into two regions - an outer region which is assumed to be essentially nondissipative, and an inner region in which viscosity is assumed to play an important role. Figure 2 expresses the separated region in terms of Crocco-Lees' language. The extent of the viscous region is measured by the length, δ, which for the case of a body in high-Reynolds-number stream is the usual boundary layer thickness. The definition of the length δ is artificial, and physical quantities such as pressure and interaction distance should not be sensitive to the definition of δ. In order that the equations describing attached, separated, and reattaching flows can be handled, the following simplifying assumptions are made:

1. The gradients of viscous or Reynolds stresses in the flow direction are negligible compared with the static-pressure gradient in the flow direction.
2. The pressure gradient transverse to the stream direction is negligible.
3. Steady flow exists.
4. The external flow is plane, isentropic, supersonic over a flat, adiabatic wall. The flow direction at $y = \delta$ is given by the Prandtl-Meyer relation.

*Numbers in parenthesis refer to references at the end of this report.

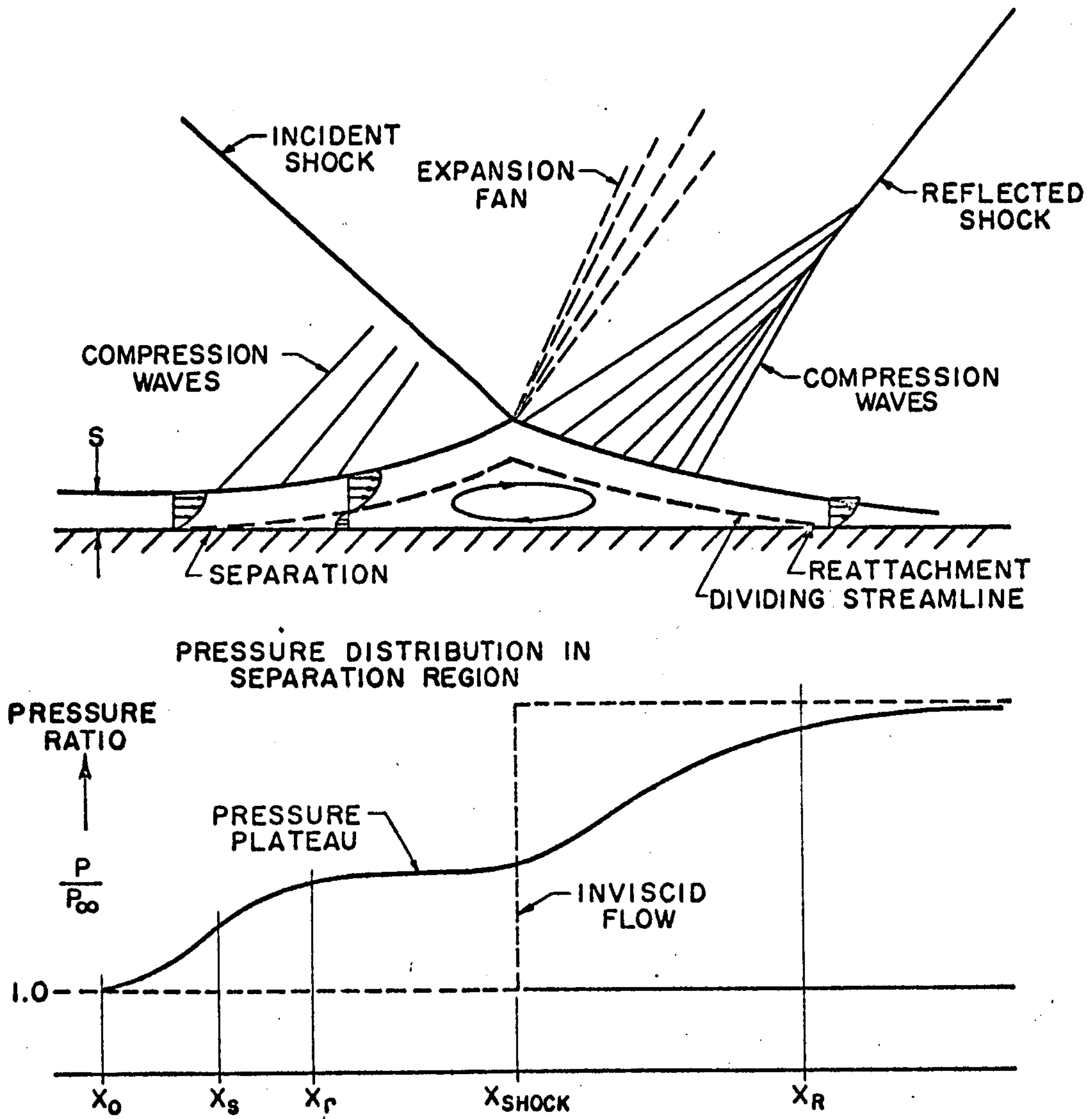

Figure 1-a. Shock Wave-Laminar Boundary Layer Interaction Model
Shock Generated by External Source

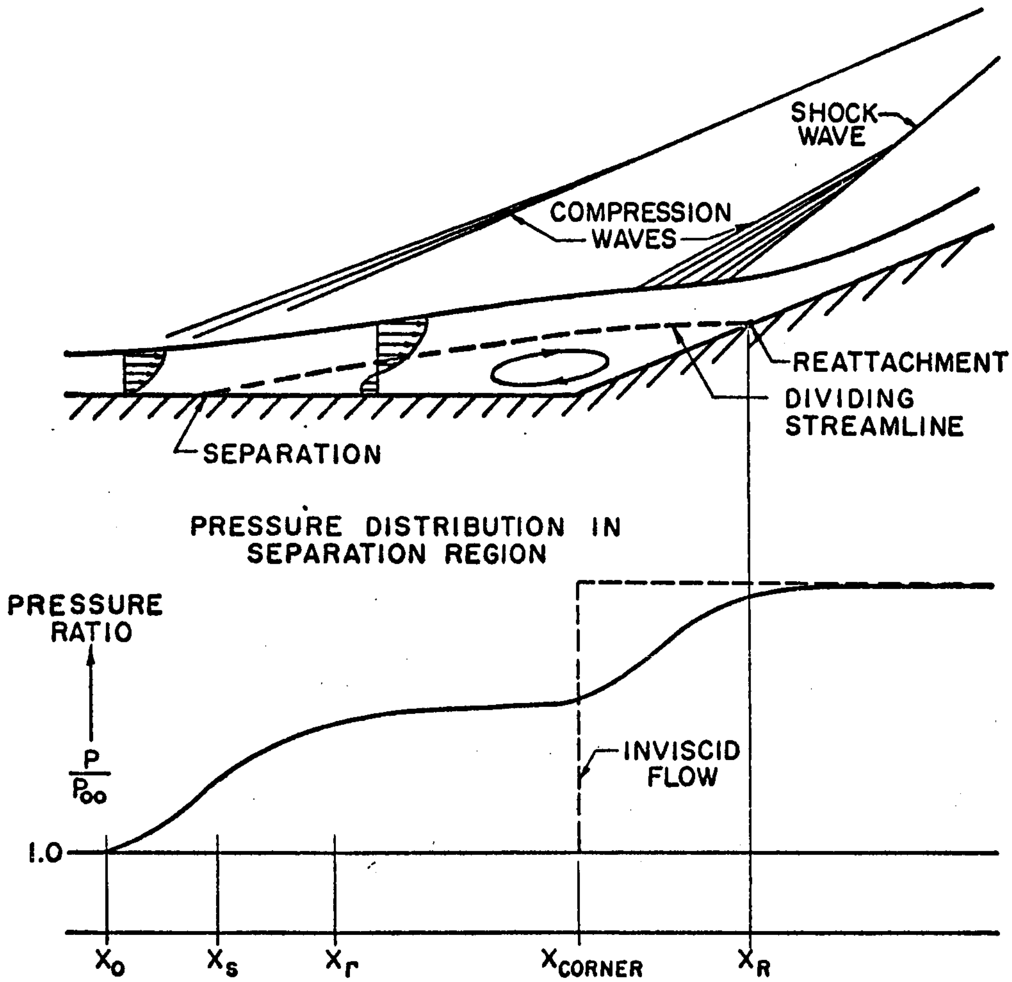

Figure 1-b. Shock Wave-Laminar Boundary Layer Interaction Model Shock Generated by Ramp

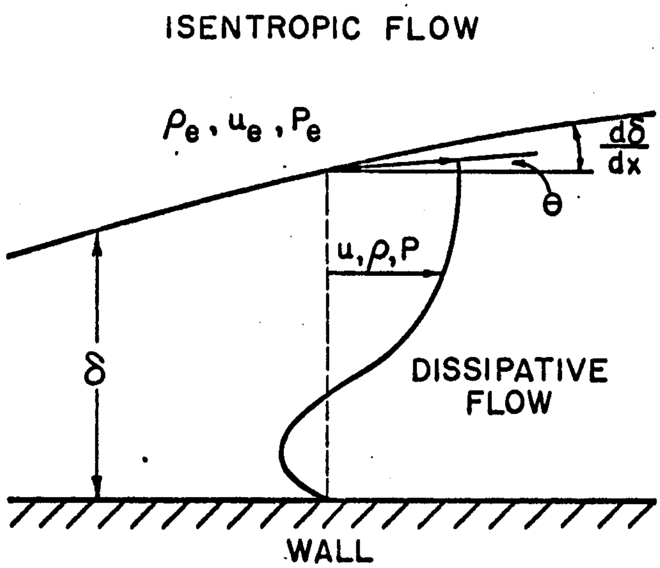

Figure 2. Flow Regions as Described by Crocco-Lees

5. Prandtl number is unity.
6. Viscosity is proportional to absolute temperature.
7. Flow angles relative to wall are small.
8. The gas is thermally and calorically perfect.
9. Stagnation temperature is constant throughout the whole flow.
10. Viscous region is laminar.

The basic parameter of the Crocco-Lees method, and the one used to characterize the flow in the viscous region is defined as K, where

$$K = \frac{\text{momentum flux}}{\text{mass flux x local external velocity}} = \frac{u_1}{u_e} . \tag{1}$$

The velocity at the edge of the boundary layer is u_e. A mean velocity in the viscous region is defined by equation (1) and is denoted by u_1. The displacement and momentum thicknesses are defined in the usual manner.

$$\delta^* = \int_0^\delta (1 - \frac{\rho u}{\rho_e u_e}) dy = \text{displacement thickness}$$

$$\delta^{**} = \int_0^\delta (\frac{\rho u}{\rho_e u_e})(1 - \frac{u}{u_e}) dy = \text{momentum thickness}$$

The basic parameter (K) defined in terms of either the compressible or incompressible boundary layer variables becomes

$$K = \frac{\delta - \delta^* - \delta^{**}}{\delta - \delta^*} = \frac{\delta_i - \delta_i^* - \delta_i^{**}}{\delta_i - \delta_i^*} . \tag{2}$$

The parameter f, appearing in the mean temperature-mean velocity relation, is defined as

$$f = \frac{(\delta_i - \delta_i^* - \delta_i^{**})\delta_i}{(\delta_i - \delta_i^*)^2} = \frac{K \delta_i}{(\delta_i - \delta_i^*)} . \tag{3}$$

The deviations of f and K from unity measure, in a sense, the nonuniformity of the velocity profile. For every incompressible boundary layer flow, f and K can be related to each other, so that the given incompressible flow and the corresponding complete family of compressible boundary layer flows obtained through the Stewartson (10) transformation are characterized by a certain f(K) relation. Each flow corresponds to a point in the f-K plane and the whole class of flows is represented by a single f-K curve.

For convenience, an alternate function of f is defined as

$$F \equiv (f/K^2) - 1 = \frac{\delta_i^* - \delta_i^{**}}{\delta_i - \delta_i^* - \delta_i^{**}} . \tag{4}$$

Similarly, since F and K are defined by incompressible boundary layer parameters, for every incompressible velocity profile there are unique values of F and K.

Glick noticed that for a given value of the form factor, $H_i = \frac{\delta_i^*}{\delta_i^{**}}$, the mean-temperature parameter, f, goes through a maximum at a finite value of δ_i, while K generally has the property that it increases montonically towards unity with increasing δ_i. By choosing δ_i such that f is maximum, one obtains a simple analytical expression for f(K), which is

$$f(K) = K^2/(2K - 1) \; ,$$
$$F(K) = 2(1 - K)/(2K - 1) \; . \tag{5}$$

The other two correlation relations necessary to complete the formulation of the Crocco-Lees method, C(K) and D(K), can be obtained by first using the Stewartson transformation to eliminate compressibility effects and then examining known incompressible solutions. In previous studies using the Crocco-Lees method, the C(K) and D(K) relations that have been used were those obtained from the Falkner-Skan solutions. The C(K) is the mixing rate correlation function and D(K) is the skin-friction correlation function. Roughly speaking, C(K) for the Falkner-Skan solution is essentially constant from separation to the Blasius flow condition, while the other theoretical solutions and the experimental Schubauer (11) ellipse data show a trend in which C(K) drops sharply going from the Blasius condition to separation. This difference is associated with the physical fact that Falkner-Skan flows are similar flows which do not have "histories" and do not reflect the essential change in shape of the velocity profile prior to separation.

II-2.1 Upstream of separation. The two-dimensional flows upstream of separation use the correlation functions obtained by the maximum f principle mentioned above, a new C(K) relation based on boundary layers that have "histories," and a D(K) relation obtained by assuming that D(K) decreases linearly from the Blasius value at $K_b = .693$ to zero at $K_s = .630$. The C(K) relation that has been chosen is one that decreases linearly from the Blasius value of (K) to zero at the separation value of K. (Glick assumed $K_b = .693$ and $K_s = .630$ to be the best approximations for Blasius flow and the separation point.) The correlation equations are:

$$C(K) = 36.2 \; (K - .630) \; ,$$
$$F(K) = 2 \; (1 - K)/(2K - 1) \; , \tag{6}$$
$$D(K) = 22.2 \; (K - .630).$$

The Crocco-Lees equations, linearized with regard to Mach number,

$$\epsilon \ll M_\infty \quad \text{and} \quad M = M_\infty + \epsilon$$

become

$$\frac{dK}{d\zeta} = -L\left[\frac{P}{\zeta} - \epsilon\right], \; \frac{d\epsilon}{d\zeta} = -N\left[\frac{Q}{\zeta} - \epsilon\right] \tag{7}$$

where ζ is a kind of local Reynolds number.

$$\zeta = \frac{m}{\mu_t a_t} = \frac{m}{\mu_t}$$

The L, N, P, Q parameters are obtained from the following equations:

$$L = \frac{2K(1-K)(2K-1)^2\sqrt{M_\infty^2-1}}{4M_\infty K(1-K)(2K^2-2K+1)(1+\frac{\gamma-1}{2}M_\infty^2)C(K)S}$$

where

$$S = \left[1 - \frac{\gamma M_\infty^2(2K-1)^2}{2(2K^2-2K+1)(1+\frac{\gamma-1}{2}M_\infty^2)} - \frac{(2K-1)^3(M_\infty^2-1)}{4(2K^2-2K+1)(1-K)(1+\frac{\gamma-1}{2}M_\infty^2)^2}\right],$$

$$N = \frac{LM_\infty}{KF(K)} = \frac{LM_\infty(2K-1)}{2K(1-K)},$$

$$P = \frac{C(K)\ M_\infty}{\sqrt{M_\infty^2-1}}\left[\sigma(1-K) + \frac{(2K+1)(1-\sigma)}{2K}\left\{\frac{2\gamma K(1-K)M_\infty^2}{(2K-1)} + K\left(\frac{\gamma+1}{2}\frac{M_\infty^2}{1+\frac{\gamma-1}{2}M_\infty^2} - 1\right)\right\} - \frac{2K(1-K)(1+\frac{\gamma-1}{2}M_\infty^2)}{(2K-1)}\right],$$

$$Q = \frac{C(K)\ M_\infty}{\sqrt{M_\infty^2-1}}\left[\sigma(1-K) - \frac{2(1-K)K(1+\frac{\gamma-1}{2}M_\infty^2)}{(2K-1)}\left\{1 - \frac{(1-\sigma)(2K^2-2K+1)}{K(2K-1)}\right\}\right],$$

$$\sigma = D(K)/2(1-K)\ C(K).$$

In calculating a separating flow problem, the free stream Mach number, M_∞, is known; thus L, N, P, and Q depend only on K. A value of ζ is chosen at the separation point, which is equivalent to selecting the value of the separation Reynolds number. Trial values of ϵ at separation are chosen, and equations (7) are numerically integrated in the upstream direction. The correct value for ϵ at separation is obtained when the integrated quantities approach the limit values of ϵ_b and ζ_b at the Blasius point ($K = .693$); that is,

$$\epsilon_b = \frac{M_\infty(1+\frac{\gamma-1}{2}M_\infty^2)C(K)(1-K)^2}{\sqrt{M_\infty^2-1}\ \sqrt{A}\ \sqrt{Re_x}}\left(1 - \frac{KF}{(1-K)}\left[1+\frac{\gamma-1}{2}M_\infty^2\right]\right)\Bigg|_{K=K_b} \qquad (8)$$

where $A = .44$, $Re_x = \frac{\rho_\infty u_\infty x}{\mu_\infty}$, and

$$\zeta_b = t\ Re_{\delta**}/(1-K) \qquad (9)$$

where

$$t = T_e/T_t,\ \text{and}\ Re_{\delta**} = \sqrt{.44}\ \sqrt{Re_x}.$$

The value of ζ is not as sensitive in the iteration as is ϵ. Once ζ_{sep} and ϵ_{sep} have been found, then the corresponding locations and pressures are found from:

$$\frac{x_s - x}{x_s} = \frac{1}{Re_{x_s}} \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^2 \int_{\zeta}^{\zeta_s} \frac{\zeta d\zeta}{C(K)} \frac{M_\infty}{M_e} \left[\frac{1 + \frac{\gamma-1}{2} M_e^2}{1 + \frac{\gamma-1}{2} M_\infty^2}\right]^{\frac{(3\gamma-1)}{2(\gamma-1)}}, \quad (10)$$

$$\frac{p}{p_o} = \left\{\frac{1 + \frac{\gamma-1}{2} (M_\infty - \epsilon_b)^2}{1 + \frac{\gamma-1}{2} M_e^2}\right\}^{\left(\frac{\gamma}{\gamma-1}\right)} . \quad (11)$$

II-2.2 Between separation and pressure plateau. The problem of separated and reattaching flows must be treated in a manner that is different from the way in which the problem up to separation was studied, since no detailed theoretical studies of separated and reattaching flows exist. After separation, the flow is essentially divided into two parts by the dividing streamline - one part includes all the fluid upstream of separation and the other part is a steady circulating flow in which the fluid elements continuously undergo a cycling action. The fluid along the dividing streamline is accelerated by viscous momentum transfer in the region between separation and the beginning of reattachment and is thereby prepared for the forthcoming reattachment pressure rise in which fluid along the dividing streamline is stagnated.

In re-examining the formulation of the Crocco-Lees method beyond separation, it became clear that in order to determine the correlation relations quantitatively experimental results must be used, since no satisfactory theoretical data are available. One particular experiment, performed at a free-stream Mach number of 2.45 and a free-stream Reynolds number per inch of 6×10^4 was selected.

The correlation functions have been determined only up to separation. D(K) is assumed to be zero since the skin friction is small in the separated region. The F(K) relation between separation and shock impingement is assumed to remain constant at the separation value. C(K) is expected to rise continuously from zero near separation to a high value upstream of shock impingement. As a simplified assumption, C(K) is taken to be a constant value ($\bar{C}$) between separation and shock impingement. (In a later "refined" attempt, Glick used two C(K) values; C_1 was a constant used between separation and the beginning of the pressure plateau, and C_2 was the value assumed during the pressure plateau.) These "C" values were obtained from the single experimental data by establishing a relation between the reattachment pressure rise and the associated length ratio ($\Delta x/x_s$). The "C" values obtained are then regarded as universal and employed in the analysis of all other separating flows.

The linearized equations which apply downstream of separation are:

$$\frac{dK}{d\zeta} = \frac{-F_s\sqrt{M_\infty^2 - 1}}{M_\infty\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)C_1 b} \left[\frac{C_1 M_\infty\left(1 + \frac{\gamma-1}{2} M_\infty^2\right)}{\zeta\sqrt{M_\infty^2 - 1}} \left(\frac{F_s^2 - (1-K)b}{F_s}\right) + \epsilon\right], \quad (12)$$

$$\frac{d\epsilon}{d\zeta} = \frac{-\sqrt{M_\infty^2 - 1}}{K(1 + \frac{\gamma-1}{2} M_\infty^2)C_1 b}\left[\frac{C_1 M_\infty (1 + \frac{\gamma-1}{2} M_\infty^2) F_s}{\zeta\sqrt{M_\infty^2 - 1}} + \epsilon\right],$$

$$b \equiv F_s^2 + \frac{\gamma F_s M_\infty^2}{(1 + \frac{\gamma-1}{2} M_\infty^2)} + \frac{(M_\infty^2 - 1)}{(1 + \frac{\gamma-1}{2} M_\infty^2)^2} . \tag{12}$$

By using the universal value for C_1 (Glick used $C_1 = 11.0$) these equations may be numerically integrated in the downstream direction. As in the previous calculations, the pressure distribution may be obtained from equations (10) and (11).

The initial assumptions 1-10 largely stemmed from attached boundary layer theory and are in question only for the separated parts of the flow. The lack of knowledge of the F(K), C(K), and D(K) relations for the separated region is considered to be the major limitation of the method.

II-3. Lees and Reeves Method (12)

The aim of this method was to construct a theory that is capable of including the entire flow within a single framework, without introducing semi-empirical features. The method employs the first moment of the momentum in addition to the usual momentum integral (zeroth moment). This method itself is not new, but it turns out that its successful application to separated and reattaching flows hinges on the proper choice of the one-parameter family of velocity profiles utilized to represent the integral properties of the viscous flow. The Stewartson (5) reversed flow profiles were found to have the qualitatively correct behavior while polynomials did not.

In order to avoid the semi-empirical features of the Crocco-Lees method for separated and reattaching flows, at least one additional moment of the momentum equation must be employed. Several other methods, and more recently the one by Tani (13), use the same approach. This application combines the attractive features of Tani's technique with the appropriate Stewartson reversed flow profiles.

Upstream of separation the interaction between the laminar boundary layer and the external supersonic flow is completely determined by the Reynolds number and the previous history of the boundary layer. Downstream of separation, the magnitude of the peak reversed-flow velocity in the viscous layer increases steadily with distance along the surface, reaches a maximum, and then decreases again as the dividing streamline moves farther and farther away from the surface. A polynomial representation of the velocity profile based on a single parameter was found to be inadequate to describe the sequence of events.

The solution for flow upstream of separation as developed in this method requires the iteration solution of two simultaneous equations in which two variables must simultaneously vanish as the Blasius condition is reached. Between separation and shock impingement the solution is obtained by solving two equations. This solution is uniquely determined by the conditions previously found for separation.

Downstream from the shock impingement point the three quantities "a" (velocity profile parameter), the local Mach number, and the transformed displacement thickness all decrease until reattachment is reached.

An interesting sidelight mentioned by Lees and Reeves is the definition of "subcritical" and "supercritical" flows. When $(d\delta/dp) > 0$ the flow is termed "subcritical," whereas the flow is "supercritical" when $(d\delta/dp) < 0$. A subcritical boundary layer is capable of generating its own positive pressure gradient in the flow direction by interacting with an external, inviscid supersonic stream. A supercritical flow, on the other hand, responds to a pressure rise generated downstream only through a sudden "jump" or "shock" to a subcritical state. Within the framework of the Crocco-Lees mixing theory, adiabatic laminar boundary layers are subcritical, whereas adiabatic turbulent boundary layers are supercritical.

II-4. Makofski Method (14)

This method uses a modified Pohlhausen approach with the velocity distribution represented by a fifth-degree polynomial with two undetermined parameters. One of the parameters is related to the skin friction at the wall while the other is proportional to the imposed pressure gradient. In the regions of flow separation the concept of the dividing streamline is introduced in order to compute the length of the separated region and the beginning of reattachment.

The method of analysis used by Makofski consists of transforming the compressible laminar boundary layer equations into incompressible form, obtaining integral relations, and finally, solving these relations by use of the fifth-degree polynomial representation of the velocity profile.

The parameters "a" and "b" which describe the velocity profiles are dependent only upon the local Mach number and Reynolds number and are independent of the agency causing the disturbance. For a flat plate without pressure gradient (Blasius flow), "a" is 1.78365 and "b" is 0. For "a" less than zero, the flow will be separated from the wall. The interaction is still described by the equations developed for the attached flow except that the concept of the dividing streamline must be introduced in order to compute the length of the separated region and the reattachment pressure rise.

Makofski compared his analytical calculations at Mach 2.0 with experimental data and in his words, the correlation was "excellent." However, this method is more complex than that described by Pinkus (see below) and it shares the same weakness; that is, it presupposes the position of the dividing streamline. Thus in its present form, the Makofski method cannot be used to predict the flow in the separated region when only the geometry and upstream flow conditions are given.

II-5. Pinkus Method (15)

A system of equations was developed by Pinkus which apply to the case of separated laminar boundary layers on compression corners and curved surfaces. This method is an extension of Tani's work. Tani had applied his approach only to attached flows while Pinkus extended this to separated flows. Both methods are based on a quartic velocity profile and make use of the moment-of-momentum boundary layer equation.

The separated boundary layer is divided into regions as shown in Figure 3. The dividing streamline is assumed to be represented by the direction of the isentropic stream, which makes the calculation of the behavior of separated flows relatively simple.

A fourth-order polynomial represents the velocity profile

$$\frac{u}{u_e} = a_0 + a\left(\frac{Y}{\Delta}\right) + b\left(\frac{Y}{\Delta}\right)^2 + c\left(\frac{Y}{\Delta}\right)^3 + d\left(\frac{Y}{\Delta}\right)^4 \qquad (13)$$

where the coefficients are determined by boundary conditions:

$$Y = \Delta: \quad u = u_e, \quad \frac{\partial u}{\partial Y} = 0, \quad \frac{\partial^2 u}{\partial Y^2} = 0$$

$$Y = 0: \quad u = 0$$

The remaining boundary condition,

$$\frac{\partial}{\partial Y}\left(\mu \frac{\partial u}{\partial Y}\right) = \rho_e u_e \frac{du_e}{dX} \quad @\ Y = 0,$$

is dropped so that the coefficients in the quartic depend in this case on "a". The arbitrary parameter "a" has physical meaning in that it is proportional to the shearing stress at the wall. When a = 0, the shear at the wall is zero and the flow is ready to separate. The reattachment point is also represented by a = 0. The usefulness of "a" lies not only in defining the region of separation but also in the fact that it extends to and embraces the attached and transition regions. Values of "a" for the constant pressure solution (dp/dx = 0) are 1.857 and -4.887. These represent the extremities for "a", the Blasius type flow when a = 1.857, and the reversed flow in the pressure plateau region where a = -4.887. Figure 3 shows the behavior of the variable "a" as the flow passes through a laminar separation region.

The analysis is broken into three regions: the detachment, central, and reattachment. The dividing streamline is significant because it commences at the point of separation and ends at the point of reattachment. As mentioned earlier, it is mathematically more convenient to postulate that the dividing streamline determines the deflection angles of the external stream.

In the detachment region a system of 3 equations and 3 unknowns is developed. By simultaneously solving these, the required boundary layer thickness and local Mach numbers are obtained; hence the pressure distribution can be obtained.

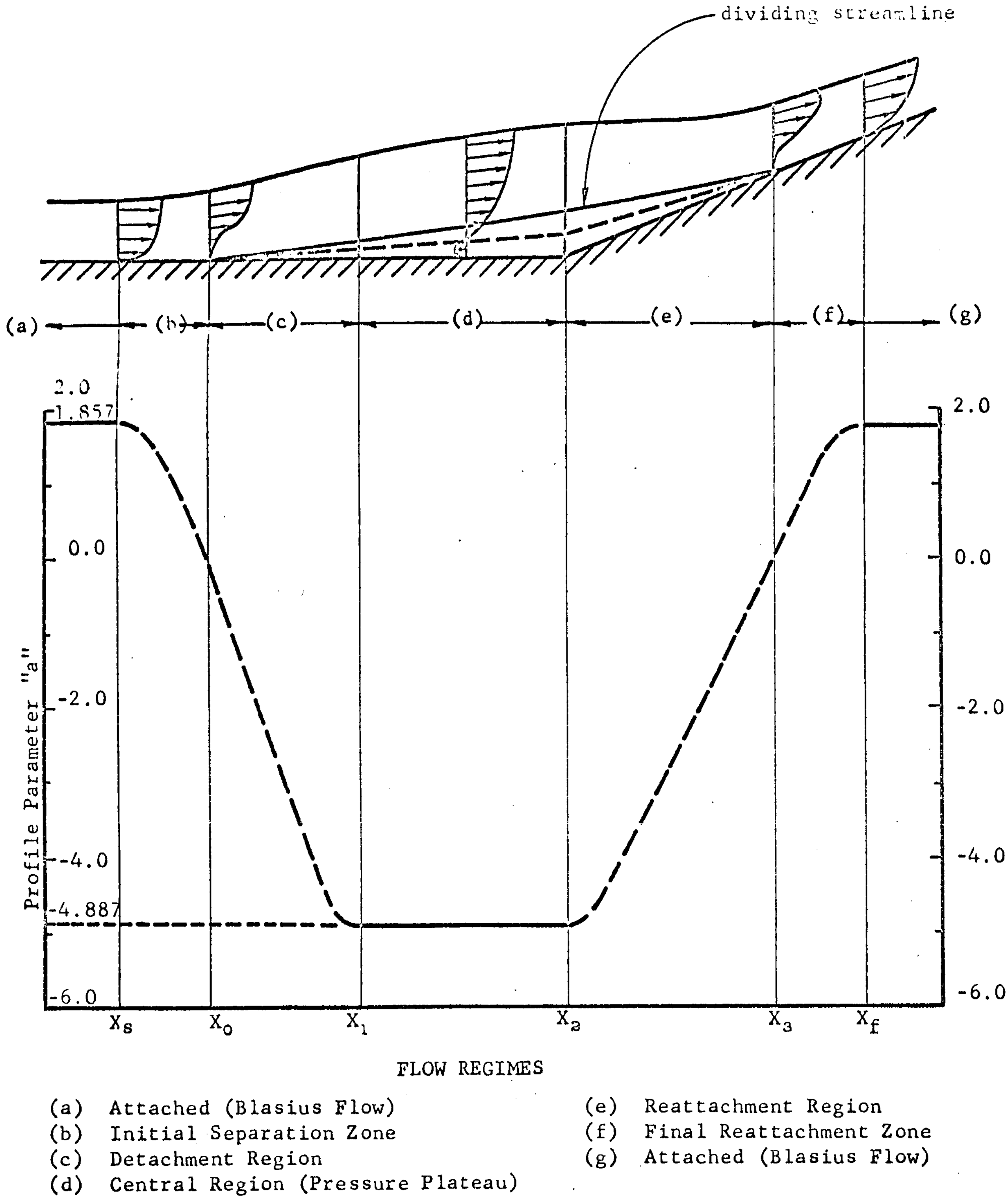

Figure 3. Pinkus Flow Model and Variation of Velocity Profile Parameter "a".

In the central region (from x_1 to x_2) the pressure is essentially constant. This means a constant Mach number and the dividing streamline has a constant slope. Again, three equations must be used to solve this region.

The onset of the reattachment region is characterized by a pronounced increase in the pressure gradient. The Prandtl-Meyer compression formula, together with the boundary layer equations, provides the solution for "a" in this region (x_2-x_3). The solution for this region is constrained because the dividing streamline must asymptotically meet the wall at the reattachment point. This is handled by a constant (K_2) that must be suitably chosen so that both the magnitude of the pressures and the length of the reattachment region are satisfied. In other words, the end conditions and the reattachment point impose the restrictions on the choice of K_2.

III. LITERATURE SURVEY (EXPERIMENTAL PRESSURE DATA)

III-1. Subsonic Flow

At the present the most valuable data available for subsonic, laminar separation correlations are that taken by G. Schubauer (11) on an elliptical cylinder. In this experiment Schubauer placed an elliptical cylinder perpendicular to an air flow of 11.5 feet per second velocity. Velocity profiles across the boundary layer were measured with a hot wire anemometer at twelve stations, from the stagnation point to just after the separation point. Also the static wall pressure was measured at sixteen points around one side of the ellipse. Thus both velocity profile and wall static pressure measurements were available for conversion into Crocco-Lees notation and subsequent correlation.

The only other applicable subsonic experimental data which has been found is that of Fage (16). A proper evaluation of the data contained in Fage's paper has not yet been carried out.

III-2. Supersonic Flow

The available pressure data for laminar separations is restricted to the lower Mach numbers. The lack of data above approximately Mach 3.0 has also been noted by others in the literature.

The investigation of the transition region by Chapman (17) presents several pressure distributions of interest at lower Mach numbers. Also, Gadd et al. (18) present pressure data in their study. These studies represent two broad experimental programs for determining the effects of various parameters on transition in the separated region.

A recent report by Pate (19) presented laminar separation results at Mach 3.0. Also of interest in this report are the velocity profiles which were measured by a pitot probe. A boundary layer thickness of approximately 0.4 inches permitted profile measurements which show the reversed flow in the separated region.

IV. IBM PROGRAM USING GLICK'S METHOD

Computer programs were written for the IBM 1410 using the Glick method as explained in Section II-2. The technique used consists of two programs: one that calculates the pressure distribution ahead of the separation point, and the second which computes values between separation and the pressure plateau.

The first program, between separation and the Blasius point, requires that the values of ϵ and ζ, analogous to Mach and Reynolds numbers, be chosen for the separation point. Once chosen, these values are used to start the step-by-step calculation of equations (7) which move upstream to the Blasius point in ΔK increments. The values at the Blasius point are known from equations (8) and (9); hence, repeated choices of ϵ and ζ at separation must be tried in order to end with the correct values at the Blasius point. The program, through repeated iterations, converges on the desired values of ϵ and ζ at separation. Once these are known, equations (10) and (11) are solved to obtain the pressure distribution.

Between separation and the pressure plateau, a second program which solves equations (12) was developed. The ϵ and ζ values found in the first program at separation are used as inputs. This program simply marches in ΔK steps between separation and the pressure plateau, calculating the corresponding pressure ratio and x-location values for each step.

These programs have been used to reproduce the pressure distribution curves which Glick is reported to have made using his method. The two cases which have been examined are the Mach 2.45 and 5.8 distributions. Figure 4 shows the correlation that was obtained for the Mach 2.45 case, and Figure 5 is for the Mach 5.8 case.

The correlation was found to be very poor for the Mach 5.8 problem. Glick is not explicit in his description of how the results he illustrates were obtained. However, it is implied that he used the same techniques as in the Mach 2.45 case; that is, the mixing correlation function C_1 is taken as a constant equal to 11.0 between separation and the pressure plateau. Our results show that this value is much too large, and that a value between 5.0 and 6.0 would produce the desired magnitude for the plateau pressure. Figure 5 shows two of the attempts that were made to match the curve given by Glick. In one case the value of $C_1 = 5.0$ was used, starting with the separation pressure ratio calculated by our program. The second curve starts at the separation pressure ratio shown by Glick, and uses the value of $C_1 = 5.0$ in advancing to the pressure plateau.

The experimental results by Pate (19) were used to check the computing technique. A computer analysis was performed for a laminar separation at Mach 3.0 and the analytical and experimental curves were found to be in good agreement. Further checks will now be directed to the cases having higher Mach numbers where discrepancies appear most likely.

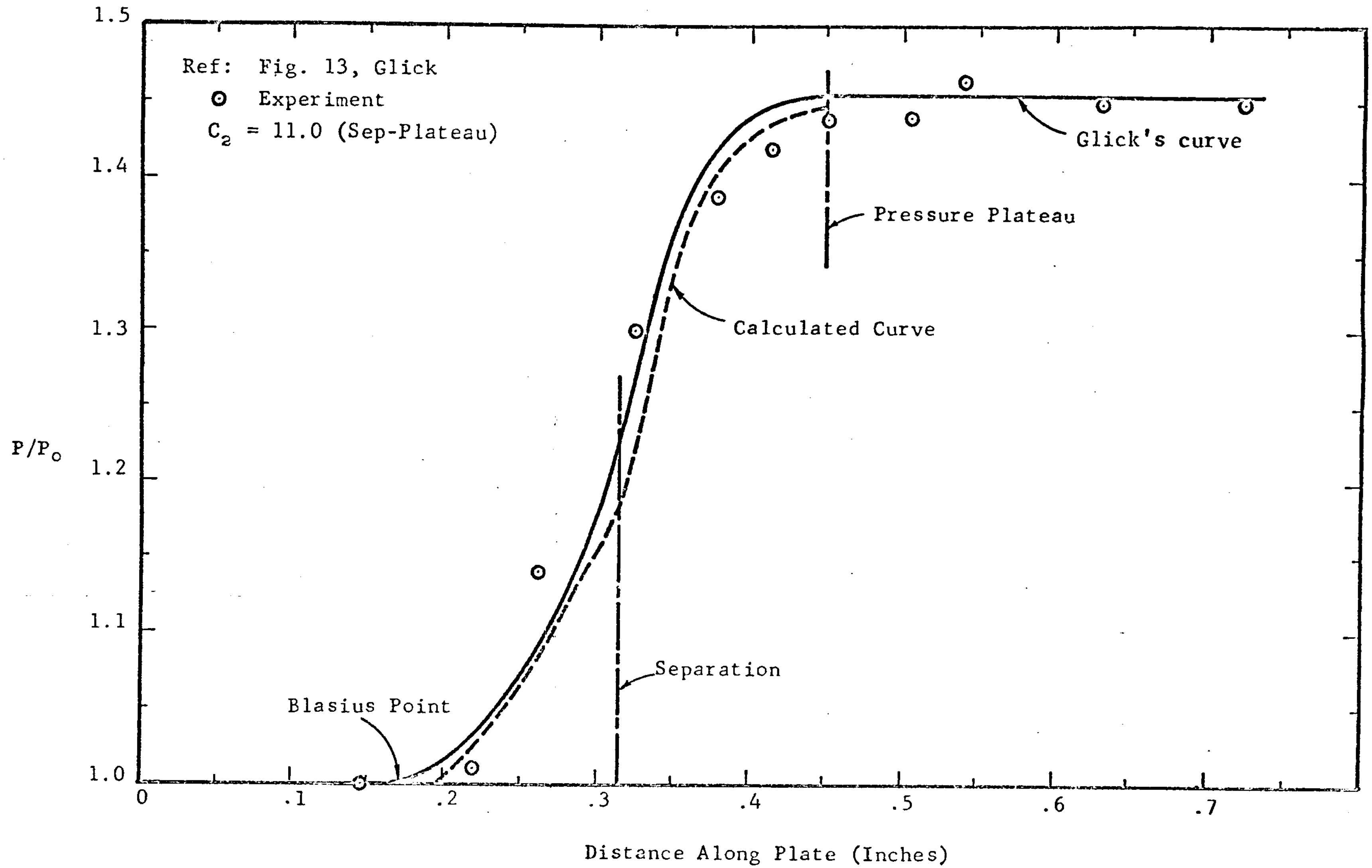

Figure 4. Pressure Distribution Along Flat Plate at $M_\infty = 2.45$

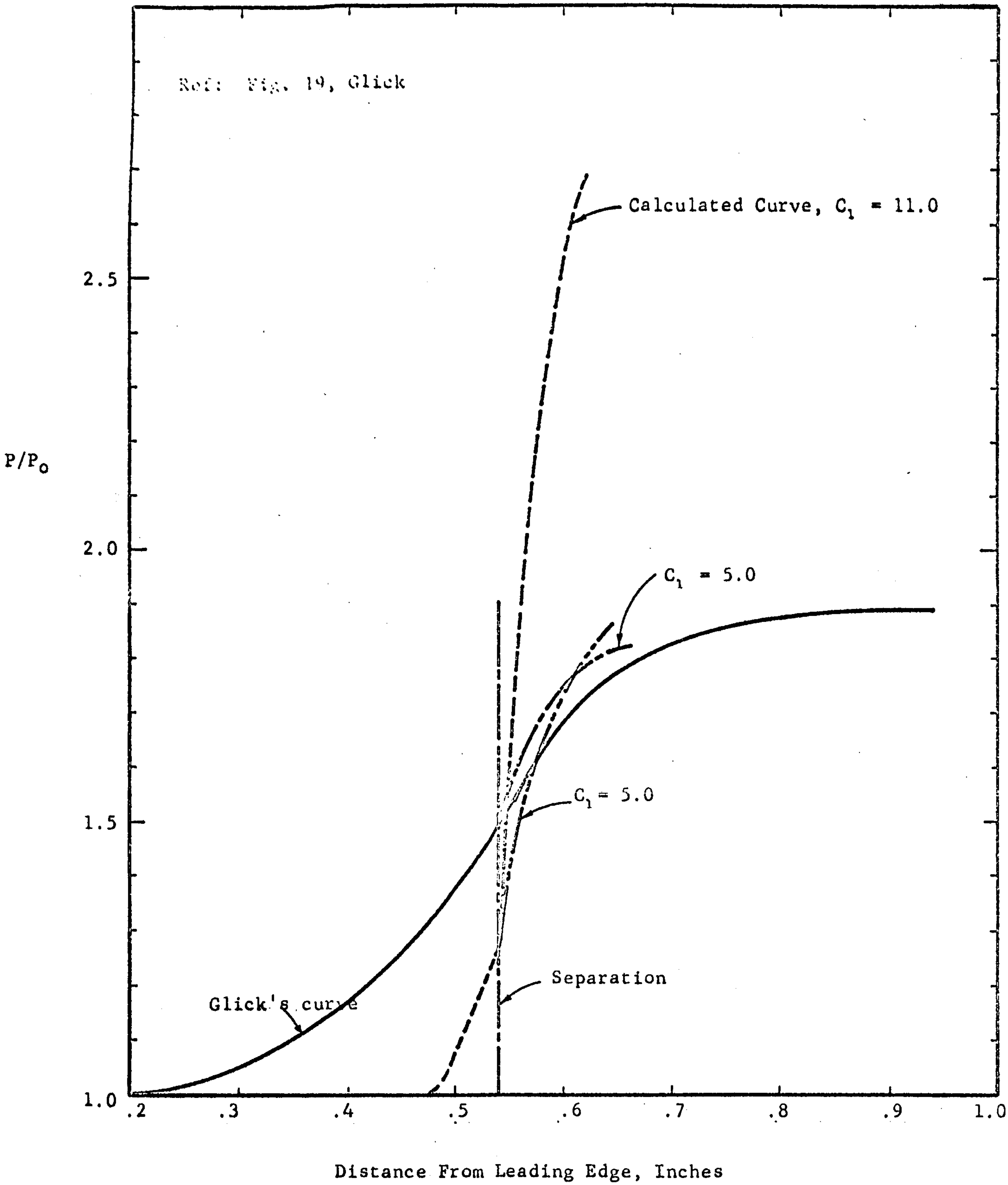

Figure 5. Pressure Distribution Along Flat Plate at M_∞ = 5.8

V. CONCLUSIONS

V-1. General Remarks

1. In spite of the long time interest in the laminar boundary layer-shock wave interaction problem, a satisfactory theoretical analysis still does not exist. With a "desired" solution, it would be possible to calculate the pressure distribution along the body and in the separated region with only the geometry and free-stream conditions known.

This problem constitutes an important class of viscous flows in which the static pressure distribution is not a given datum of the problem, but is determined by the interaction between the "external" inviscid flow and the viscous layer near the surface. The large number of variables involved in the description of the flow separation has created a formidable obstacle to the development of a quantitative theory.

2. The analytical attempts used thus far have been restricted to trying to match experimental data. That is to say, in order for the present analytical methods to generate a pressure distribution, one must know something about the actual flow field; for example, specifically where does separation occur and where does the flow reattach? Given these "additional" pieces of information, the present techniques are capable (within limitations) of generating a pressure distribution along the surface which resembles the data found experimentally. Most methods apply only in the region ahead of the incident shock on the surface. However, Pinkus has extended his formulation so that the flow up to reattachment can be conveniently handled. The Pinkus approach does not produce a "desired" solution to the problem since the pressure distribution during reattachment depends on the constant (K_2), which in turn depends on the location of reattachment.

3. The separation point must move upstream as the over-all pressure ratio is increased. This is due to two factors: 1) the separation pressure rise increases as the separation Reynolds number decreases, and 2) as the distance between the separation and shock impingement is increased, the energy of the fluid particles along the dividing streamline is generally increased, thus making it possible to support a larger reattachment pressure rise. The location of the separation point is intimately connected with the various pressure rises, and the flow responds chiefly to an over-all pressure ratio by properly adjusting the position of the separation point.

4. The pressure distribution between the Blasius flow and the separation point does not depend on the downstream shape of the body. The ratio of the pressure at separation to the pressure upstream of separation appears to be a function of Mach number and Reynolds number only, independent of shock strength.

5. Present methods are not applicable in solving the practical problems which face the design engineer because the necessary information to start the solutions is not known. Unless some rule of thumb for predicting the size of the separated region is used, there is no way to apply the techniques which represent the current state-of-the-art.

The above remarks represent statements of a general nature. The remainder of this section is devoted to a discussion of specific items.

V-2. C(K) Variations and Its Effect on Glick's Method

As noted previously, the lack of knowledge of the correlation functions C(K), F(K), and D(K) represents the major limitation in Glick's method.

The effect of these functions on the pressure distribution are quite noticeable, as may be observed in Figure 5. The IBM programs described in Section IV have the flexibility of being able to vary C(K). All computations to date have used the linear correlation functions of equations (6) in the Blasius point to separation region. It may be found later that expressions other than linear will result in better comparison with experiment.

In the separation to pressure plateau region, the pressure ratio increases as C_1 is increased, as would be expected. A constant value in this region is only a "first" approximation. As more experimental data becomes available, functional relations for C_1 may result in better "universal" values.

Much remains to be learned about the interplay of the correlation functions in Glick's method. This knowledge hinges on being able to assemble enough pressure distribution data. This data should cover broad Mach and Reynolds number ranges in order to result in "universal" correlations.

V-3. Normal Pressure Gradient

One of the most important results of the Schubauer Ellipse investigation was the discovery of a normal pressure gradient in the separation region. Figure 6 shows a comparison of the wall pressure, measured by 16 static pressure orifices with the pressure distribution at the outer edge of the boundary layer, calculated from Bernoulli's equation using the u_e velocities taken from Schubauer's velocity profiles. The agreement is reasonably good except in the region where flow initially separates, indicating that a normal pressure gradient is involved in the separation process.

The significance of the normal pressure gradient becomes obvious when one considers supersonic laminar flow on a flat plate with an impinging oblique shock wave. Inviscid theory indicates that a disturbance cannot propagate upstream; however, if the conditions are right to cause separation, then the separation will occur upstream of the impinging shock. This upstream separation is of course due to the propagation of pressure, from behind the shock wave, upstream of the shock wave through the subsonic boundary layer. Thus a normal pressure gradient exists at the point of separation and plays a significant part in the separation process.

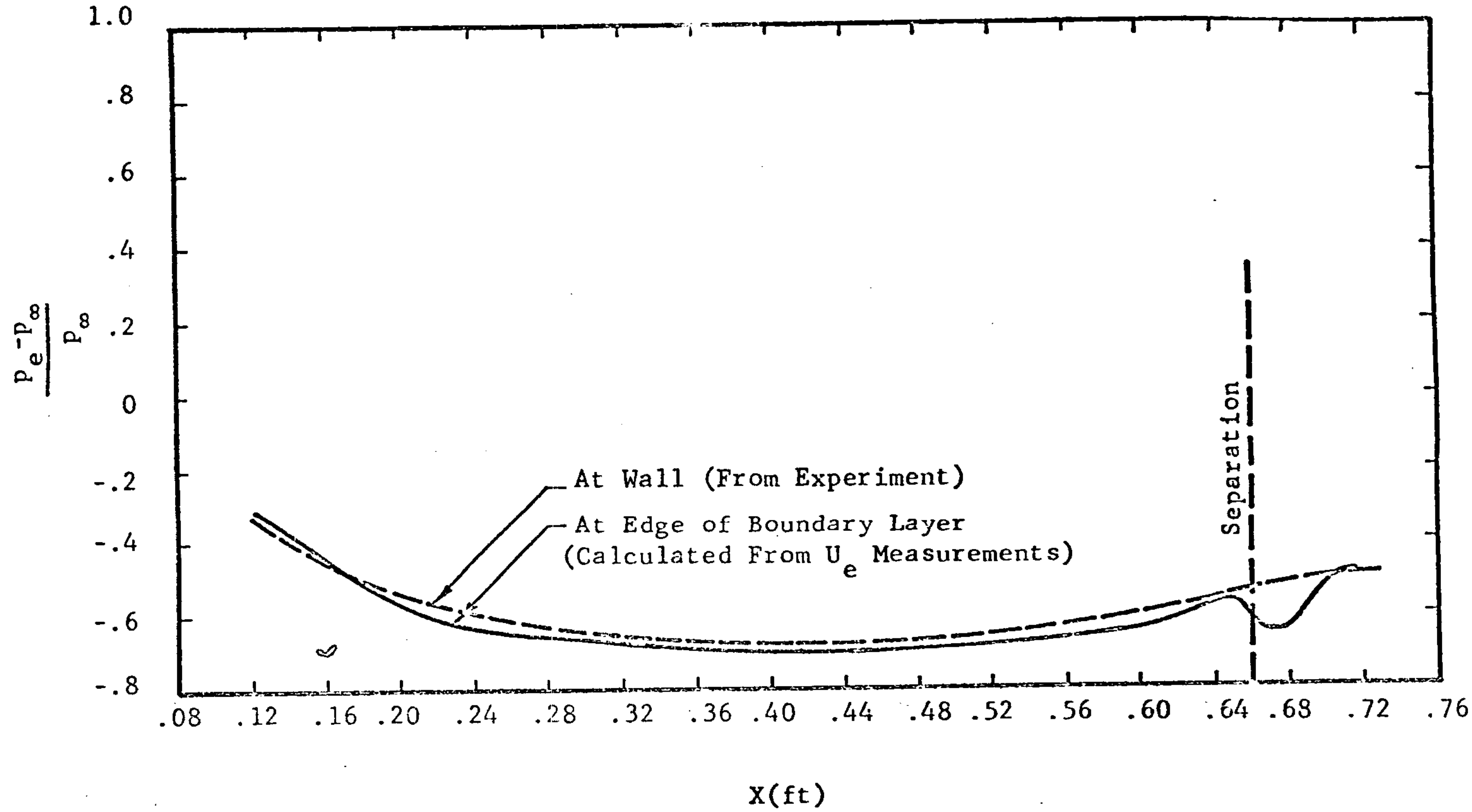

Figure 6. Pressure Distributions on Schubauer Ellipse

VI. GUIDE LINES FOR FUTURE WORK

Effort thus far has been principally devoted to: 1) surveying the literature for analytical solutions, 2) programming of Glick's method for IBM solution, and 3) collecting and correlating experimental separation data. Future work on this project will be directed towards the problem areas which are described below.

VI-1. Search for Experimental Pressure Data

The need for more experimental data is the most serious problem currently faced. Laminar separation data above Mach 3.0 has not been located.

A letter is being sent to a number of research facilities which are engaged in supersonic and hypersonic testing. It is hoped that this will produce data that will aid this study.

VI-2. Two-Dimensional Analysis

Throughout the course of this work, evidence has been accumulating which indicates that the one-dimensional approach may not be sufficient to explain the separation phenomenon in either subsonic or supersonic flow. Foremost among this evidence is the existence of a normal pressure gradient in the region where the flow separates. Momentum considerations indicate that there must be a significant normal component of momentum transfer in order to support this pressure gradient. Figure 7, showing the variation of the entrained mass flux within the boundary layer on the Schubauer Ellipse, is a typical example of the abrupt variations which the various boundary layer parameters undergo in the separation region. The difficulties inherent in attempting to explain variations of this sort in the light of present one-dimensional theory are apparent.

Quite possibly the combination of the variable quartic velocity profiles of Tani and a two-dimensional analysis could lead to an improved understanding of the role of the normal pressure gradient in the separation process. It is therefore felt that a brief analytical investigation is in order to determine more fully the advantages and disadvantages of a two-dimensional analysis of the separating boundary layer. Such an investigation might also provide hints as to how the validity of the one-dimensional approach might also be checked experimentally.

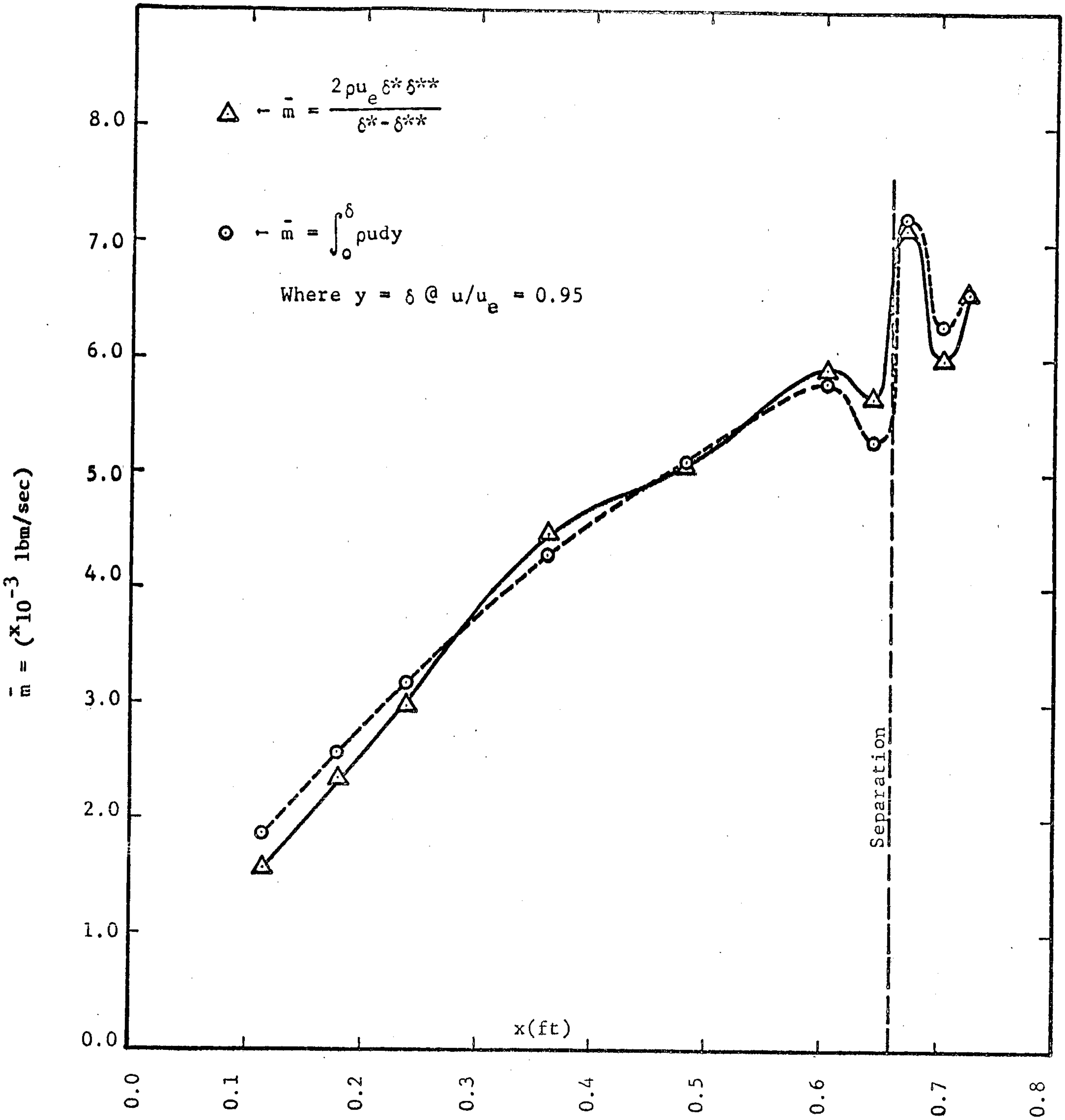

Figure 7. Boundary Layer Mass Flow Rate vs. Position on Schubauer Ellipse: Calculated by Two Methods, One Dependent and the Other Independent of Glick's F(K) Maximum Correlation

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NOMENCLATURE

a	velocity profile parameter, speed of sound
b	velocity profile parameter
$C(K)$	mixing rate correlation function
$\bar{C}$, C_1, C_2	average values of $C(K)$
$D(K)$	skin friction correlation function
f	defined in equation (3)
F	defined in equation (4)
H	form factor
K_a	Pinkus parameter
L,N,P,Q	arbitrary parameters in equations (7)
$\bar{m}$	mass flux in the x-direction $= \int_0^\delta \rho\, u\, dy$
m	$\bar{m}\, a_t$
M	Mach number
p	pressure
Re	Reynolds number
u	velocity in x-direction
U	transformed velocity u
x	coordinates along surface
Y	transformed normal coordinate
γ	ratio of specific heats
Δ	transformed δ
δ	boundary layer thickness
δ^*	displacement thickness
δ^{**}	momentum thickness
ϵ	$M_e - M_\infty$
ζ	$m/\mu_t a_t$

K	Crocco-Lees velocity profile parameter
μ	coefficient of viscosity
ρ	density

Subscripts

b	Blasius flat plate conditions
e	conditions at $y = \delta$
i	incompressible conditions
s	conditions at separation
t	free stream stagnation conditions
x	at location x
1	mean value of viscous region
∞	free stream conditions